



ENGENHARIA

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Ferramentas computacionais:

Lindo

AULA 5

PESQUISA OPERACIONAL

**Ferramentas computacionais:
Solver Excel e Lindo**

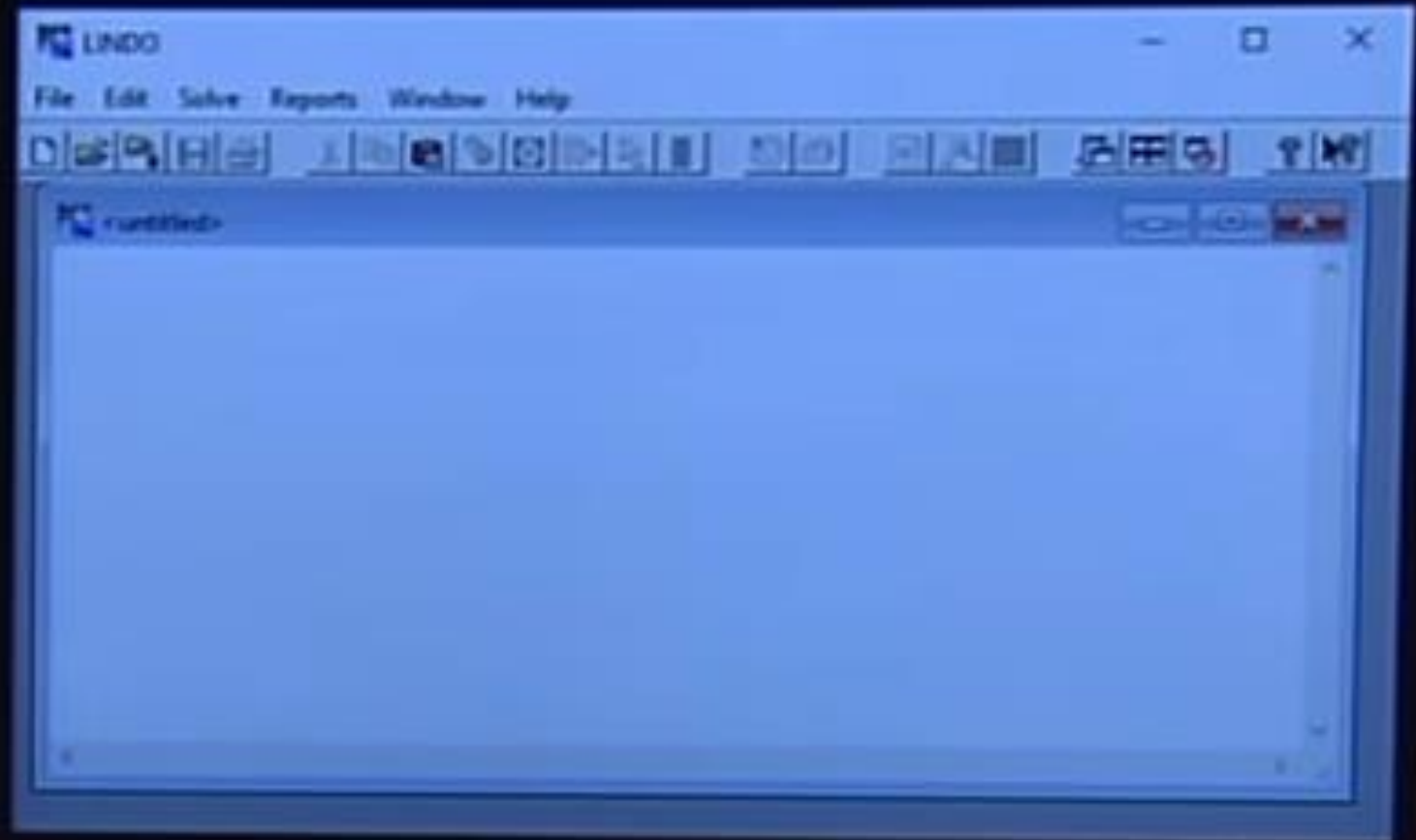
OBJETIVO

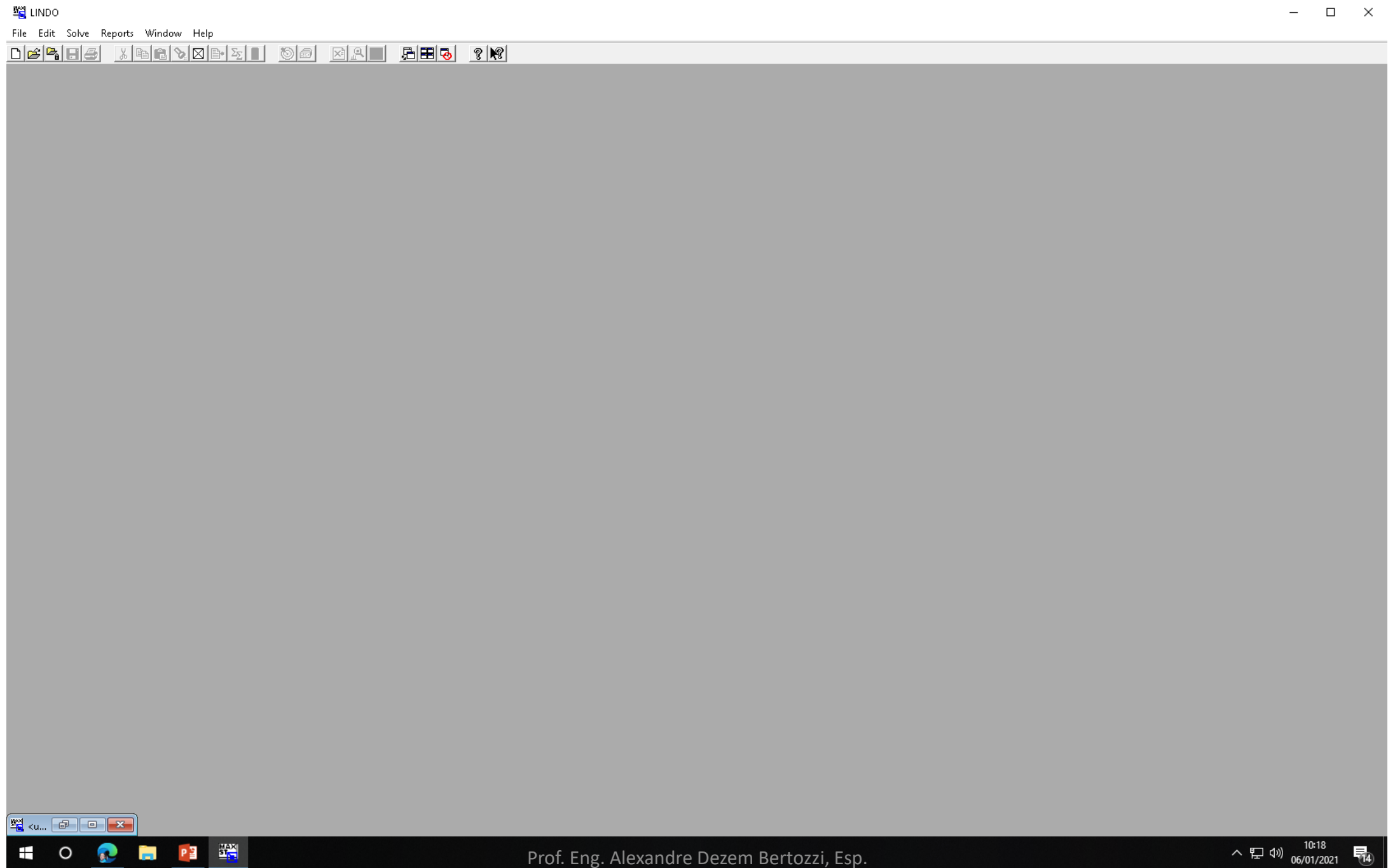
- Entender e implementar os modelos de PL e obter a solução ótima em ferramentas computacionais

LINDO

É uma IDE para desenvolvimento e resolução de problemas de programação:

- Linear
- Quadrática
- Inteira





IMPLEMENTAÇÃO

Comandos principais:

- **MAX** – Para uma função objetivo de maximização
- **MIN** - Para uma função objetivo de minimização
- **END** – Ao final do modelo e antes de definir o domínio das variáveis.

Operadores:

<	>	<=	>=
Menor	Maior	Menor ou igual	Maior ou igual

Problema 1 da Aula 3 : Determinação do *mix* de produção

$$\text{Max } z = 4X_A + 2X_B + 3X_C$$

Sujeito a

$$7X_A + 3X_B + 6X_C \leq 150$$

$$4X_A + 4X_B + 5X_C \leq 200$$

$$X_A \geq 0, X_B \geq 0, X_C \geq 0$$

P/Lindo

$$\text{Max } 4X_A + 2X_B + 3X_C$$

Subject to

$$7X_A + 3X_B + 6X_C \leq 150$$

$$4X_A + 4X_B + 5X_C \leq 200$$

 LINDO

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 <untitled>

```
Max 4xa + 2xb + 3xc
Subject to
    7xa + 3xb + 6xc <= 150
    4xa + 4xb + 5xc <= 200
```

LINDO

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<untitled>

Max $4x_a + 2x_b + 3x_c$

Subject to

$7x_a + 3x_b + 6x_c \leq 150$

$4x_a + 4x_b + 5x_c \leq 200$



<untitled>

```
Max 4xa + 2xb + 3xc
Subject to
    7xa + 3xb + 6xc <= 150
    4xa + 4xb + 5xc <= 200
```

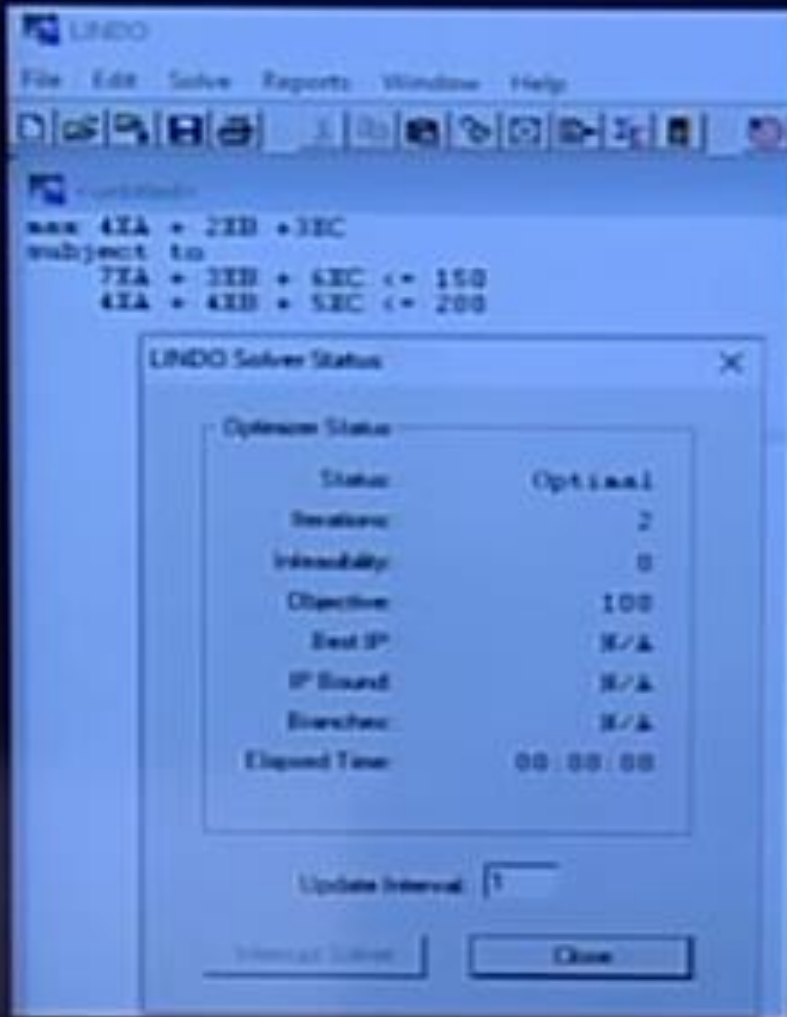
LINDO Solver Status

Optimizer Status

Status:	Optimal
Iterations:	2
Infeasibility:	0
Objective:	100
Best IP:	N/A
IP Bound:	N/A
Branches:	N/A
Elapsed Time:	00 : 00 : 00

Update Interval:

Problema 1 da Aula 3 : Determinação do *mix* de produção



LP OPTIMUM FOUND AT STEP 2
OBJECTIVE FUNCTION VALUE

1) 100.0000

VARIABLE	VALUE	REDUCED COST
XA	0.000000	0.000000
XB	50.000000	0.000000
XC	0.000000	0.625000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.500000
3)	0.000000	0.125000

NO. ITERATIONS= 2



Reports Window

LP OPTIMUM FOUND AT STEP 2 } (a)

OBJECTIVE FUNCTION VALUE

1) 100.0000 } (b)

VARIABLE	VALUE	REDUCED COST
XA	0.000000	0.000000
XB	50.000000	0.000000
XC	0.000000	0.625000

} (c)

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.500000
3)	0.000000	0.125000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
XA	4.000000	0.666667	1.111111
XB	2.000000	2.000000	0.285714
XC	3.000000	0.625000	INFINITY

ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	150.000000	200.000000	0.000000
3	200.000000	0.000000	114.285713

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 100.0000

VARIABLE	VALUE	REDUCED COST
XA	0.000000	0.000000
XB	50.000000	0.000000

LINDO Solver Status

Optimizer Status

Status:	Optimal
Iterations:	2
Infeasibility:	0
Objective:	100
Best IP:	N/A
IP Bound:	N/A
Branches:	N/A
Elapsed Time:	00:00:00

Update Interval: 1

Interrupt Solver

Close

Em Resumo teremos:

a)

b)

c)

Problema 2 da Aula 3 : Seleção de Mídia para Propaganda

UNDO

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<untitled>

Max $400000X_1 + 900000X_2 + 500000X_3 + 200000X_4$

subject to

$40000X_1 + 75000X_2 + 30000X_3 + 15000X_4 \leq 800000$

$300000X_1 + 400000X_2 + 200000X_3 + 100000X_4 \geq 2000000$

$40000X_1 + 75000X_2 \leq 500000$

$X_1 \geq 3$

$X_2 \geq 2$

$X_3 \geq 5$

$X_3 \leq 10$

$X_4 \geq 5$

$X_4 \leq 10$

LP OPTIMUM FOUND AT **STEP 0**
OBJECTIVE FUNCTION VALUE

1) 1096000000

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000

Alun@s, inserirem descrições do problema :

Problema 2 da Aula 3 : Seleção de Mídia para Propaganda

LINDO

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<untitled>

```
Max 400000X1 + 900000X2 + 500000X3 + 200000X4

Subject to
40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000
300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000
40000X1 + 75000X2 <= 500000

X1 >= 3
X2 >= 2
X3 >= 5
X3 <= 10
X4 >= 5
X4 <= 10
|
```

LP OPTIMUM FOUND AT STEP 0
OBJECTIVE FUNCTION VALUE

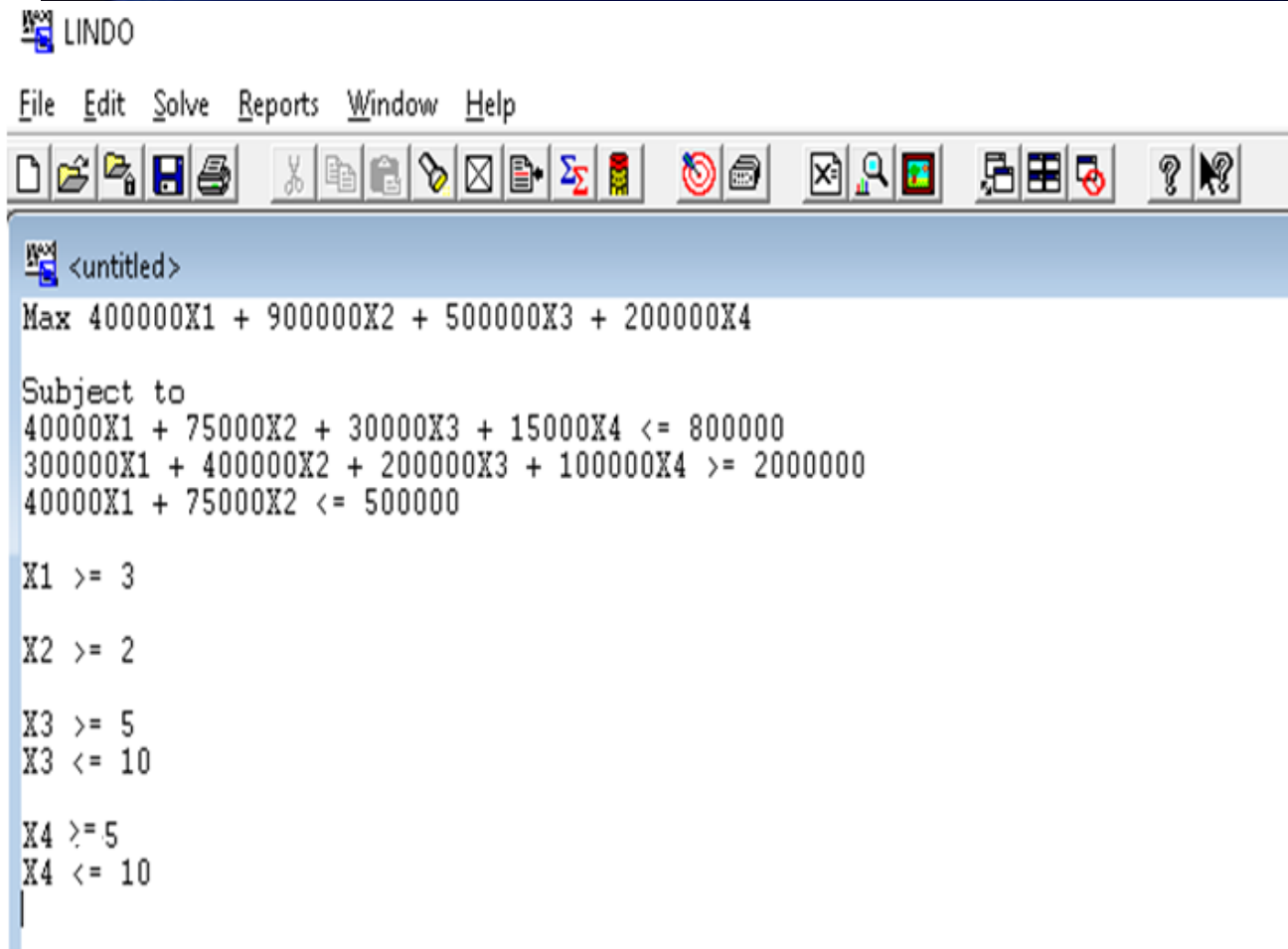
1) 1096000000

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000

Cuidados com a sintaxe

- Restrições de domínio das variáveis podem ser adicionadas ou retiradas fazendo o uso das seguintes palavras reservadas após 'END':
- INT <variável> - Domínio dos números inteiro binário.
- GIN <variável> - Domínio dos números inteiros.
- FREE <variável> - Permite que as variáveis assumam valores negativos (por padrão no Lindo, as variáveis são não negativas).

Solução Inteira:



The screenshot shows the LINDO software window. The title bar reads "LINDO". The menu bar includes "File", "Edit", "Solve", "Reports", "Window", and "Help". The toolbar contains various icons for file operations and solving. The main text area displays the following model:

```
<untitled>
Max 400000X1 + 900000X2 + 500000X3 + 200000X4

Subject to
40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000
300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000
40000X1 + 75000X2 <= 500000

X1 >= 3
X2 >= 2
X3 >= 5
X3 <= 10
X4 >= 5
X4 <= 10
|
```

LP OPTIMUM FOUND AT STEP 35
OBJECTIVE FUNCTION VALUE

1) 1080000000

VARIABLE	VALUE
X1	5.000000
X2	2.000000
X3	10.000000
X4	10.000000

Solução Inteira:

LINDO

File Edit Solve Reports Window Help



<untitled>

Max 400000X1 + 900000X2 + 500000X3 + 200000X4

Subject to

40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000

300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000

40000X1 + 75000X2 <= 500000

X1 >= 3

X2 >= 2

X3 >= 5

X3 <= 10

X4 >= 5

X4 <= 10

END

GIN 4

LP OPTIMUM FOUND AT STEP 35
OBJECTIVE FUNCTION VALUE

1) 1080000000

VARIABLE	VALUE
X1	5.000000
X2	2.000000
X3	10.000000
X4	10.000000



<untitled>

$$\text{Max } 400000X_1 + 900000X_2 + 500000X_3 + 200000X_4$$

Subject to

$$40000X_1 + 75000X_2 + 30000X_3 + 15000X_4 \leq 800000$$
$$300000X_1 + 400000X_2 + 200000X_3 + 100000X_4 \geq 2000000$$
$$40000X_1 + 75000X_2 \leq 500000$$
$$X_1 \geq 3$$
$$X_2 \geq 2$$
$$X_3 \geq 5$$
$$X_3 \leq 10$$
$$X_4 \geq 5$$
$$X_4 \leq 10$$

END

GIN 4

LINDO Solver Status

Optimizer Status

Status:	Optimal
Iterations:	38
Infeasibility:	0
Objective:	1.08e+007
Best IP:	1.08e+007
IP Bound:	1.09e+007
Branches:	1
Elapsed Time:	00:00:00

Update Interval:

Interrupt Solver

Close



Reports Window

LP OPTIMUM FOUND AT STEP 8
OBJECTIVE VALUE = 10960000.0

SET X2 TO <= 2 AT 1, BND= 0.1080E+08 TWIN= 0.1090E+08 38

NEW INTEGER SOLUTION OF 10800000.0 AT BRANCH 1 PIVOT 38

BOUND ON OPTIMUM: 0.1090000E+08

DELETE X2 AT LEVEL 1

ENUMERATION COMPLETE. BRANCHES= 1 PIVOTS= 38

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 0.1080000E+08

VARIABLE	VALUE	REDUCED COST
X1	5.000000	-400000.000000
X2	2.000000	-900000.000000
X3	10.000000	-500000.000000
X4	10.000000	-200000.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	3300000.000000	0.000000
4)	150000.000000	0.000000
5)	2.000000	0.000000
6)	0.000000	0.000000
7)	5.000000	0.000000
8)	0.000000	0.000000
9)	5.000000	0.000000
10)	0.000000	0.000000

NO. ITERATIONS= 38

BRANCHES= 1 DETERM.= 1.000E 0

LINDO Solver Status

Optimizer Status

Status:	Optimal
Iterations:	38
Infeasibility:	0
Objective:	1.08e+007
Best IP:	1.08e+007
IP Bound:	1.09e+007
Branches:	1
Elapsed Time:	00:00:00

Update Interval:

1

Interrupt Solver

Close



Reports Window

LP OPTIMUM FOUND AT STEP 38 } (*)
 OBJECTIVE VALUE = 10960000.0
 SET X2 TO <= 2 AT 1, BND= 0.1080E+08 TWIN= 0.1090E+08 38
 NEW INTEGER SOLUTION OF 10800000.0 AT BRANCH 1 PIVOT 38
 BOUND ON OPTIMUM: 0.1090000E+08
 DELETE X2 AT LEVEL 1
 ENUMERATION COMPLETE. BRANCHES= 1 PIVOTS= 38

LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 0.1080000E+08

VARIABLE	VALUE	REDUCED COST
X1	5.000000	-400000.000000
X2	2.000000	-900000.000000
X3	10.000000	-500000.000000
X4	10.000000	-200000.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	3300000.000000	0.000000
4)	150000.000000	0.000000
5)	2.000000	0.000000
6)	0.000000	0.000000
7)	5.000000	0.000000
8)	0.000000	0.000000
9)	5.000000	0.000000
10)	0.000000	0.000000

NO. ITERATIONS= 38
 BRANCHES= 1 DETERM.= 1.000E 0

LINDO Solver Status

Optimizer Status

Status: Optimal
 Iterations: 38
 Infeasibility: 0
 Objective: 1.08e+007
 Best IP: 1.08e+007
 IP Bound: 1.09e+007
 Branches: 1
 Elapsed Time: 00:00:00

Update Interval: 1

Interrupt Solver

Close

Problema 2 da Aula 3 : Seleção de Mídia para Propaganda

```
LINDO

File Edit Solve Reports Window Help

<untitled>
Max 400000X1 + 900000X2 + 500000X3 + 200000X4
subject to
  40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000
  300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000
  40000X1 + 75000X2 <= 500000
  X1 >= 3
  X2 >= 2
  X3 >= 5
  X3 <= 10
  X4 >= 5
  X4 <= 10
```

LP OPTIMUM FOUND AT STEP 0
OBJECTIVE FUNCTION VALUE

1) 1096000000

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000

LINDO Solução Inteira

File Edit Solve Reports Window Help



Reports Window

LP OPTIMUM FOUND AT STEP 38 } (*)
OBJECTIVE VALUE = 10960000.0

SET X2 TO <= 2 AT 1, BND= 0.1080E+08 TWIN= 0.1090E+08 38

NEW INTEGER SOLUTION OF 10800000.0 AT BRANCH 1 PIVOT 38

BOUND ON OPTIMUM: 0.1090000E+08

DELETE X2 AT LEVEL 1

ENUMERATION COMPLETE. BRANCHES= 1 PIVOTS= 38

LAST INTEGER SOLUTION IS THE BEST FOUND

RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 0.1080000E+08

VARIABLE	VALUE	REDUCED COST
X1	5.000000	-400000.000000
X2	2.000000	-900000.000000
X3	10.000000	-500000.000000
X4	10.000000	-200000.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	3300000.000000	0.000000
4)	150000.000000	0.000000
5)	2.000000	0.000000
6)	0.000000	0.000000
7)	5.000000	0.000000
8)	0.000000	0.000000
9)	5.000000	0.000000
10)	0.000000	0.000000

NO. ITERATIONS= 38

BRANCHES= 1 DETERM.= 1.000E 0

LINDO Solver Status

Optimizer Status

Status:	Optimal
Iterations:	38
Infeasibility:	0
Objective:	1.08e+007
Best IP:	1.08e+007
IP Bound:	1.09e+007
Branches:	1
Elapsed Time:	00:00:00

Update Interval: 1

Interrupt Solver
Close

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000

antes

VARIABLE	VALUE
X1	5.000000
X2	2.000000
X3	10.000000
X4	10.000000

depois

Problema 2 da Aula 3 : Seleção de Mídia para Propaganda

```
LINDO

File Edit Solve Reports Window Help

<untitled>
Max 400000X1 + 900000X2 + 500000X3 + 200000X4
subject to
  40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000
  300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000
  40000X1 + 75000X2 <= 500000
  X1 >= 3
  X2 >= 2
  X3 >= 5
  X3 <= 10
  X4 >= 5
  X4 <= 10
```

LP OPTIMUM FOUND AT **STEP 0**
OBJECTIVE FUNCTION VALUE

1) 1096000000

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000

Problema 2 da Aula 3 : Seleção de Mídia para Propaganda

```
LINDO

File Edit Solve Reports Window Help

<untitled>
Max 400000X1 + 900000X2 + 500000X3 + 200000X4
subject to
  40000X1 + 75000X2 + 30000X3 + 15000X4 <= 800000
  300000X1 + 400000X2 + 200000X3 + 100000X4 >= 2000000
  40000X1 + 75000X2 <= 500000
  X1 >= 3
  X2 >= 2
  X3 >= 5
  X3 <= 10
  X4 >= 5
  X4 <= 10
```

LP OPTIMUM FOUND AT **STEP 0**
OBJECTIVE FUNCTION VALUE
1) 1096000000

VARIABLE	VALUE
X1	3.000000
X2	3.066667
X3	10.000000
X4	10.000000



Reports Window

LP OPTIMUM FOUND AT STEP 38 } (*)
 OBJECTIVE VALUE = 10960000.0
 SET X2 TO <= 2 AT 1, BND= 0.1080E+08 TWIN= 0.1090E+08 38
 NEW INTEGER SOLUTION OF 10800000.0 AT BRANCH 1 PIVOT 38
 BOUND ON OPTIMUM: 0.1090000E+08
 DELETE X2 AT LEVEL 1
 ENUMERATION COMPLETE. BRANCHES= 1 PIVOTS= 38

LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 0.1080000E+08

VARIABLE	VALUE	REDUCED COST
X1	5.000000	-400000.000000
X2	2.000000	-900000.000000
X3	10.000000	-500000.000000
X4	10.000000	-200000.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	3300000.000000	0.000000
4)	150000.000000	0.000000
5)	2.000000	0.000000
6)	0.000000	0.000000
7)	5.000000	0.000000
8)	0.000000	0.000000
9)	5.000000	0.000000
10)	0.000000	0.000000

NO. ITERATIONS= 38
 BRANCHES= 1 DETERM.= 1.000E 0

LINDO Solver Status X

Optimizer Status

Status:	Optimal
Iterations:	38
Infeasibility:	0
Objective:	1.08e+007
Best IP:	1.08e+007
IP Bound:	1.09e+007
Branches:	1
Elapsed Time:	00:00:00

Update Interval:

Interrupt Solver
Close

Cuidados com a sintaxe

- Não é possível fazer uso dos parênteses () como indicadores de ordem de precedência. Todas as operações são executadas da esquerda para a direita.
- No lado direito das restrições utiliza-se constantes.
- No lado esquerdo das restrições, somente variáveis e o coeficiente da variável.

